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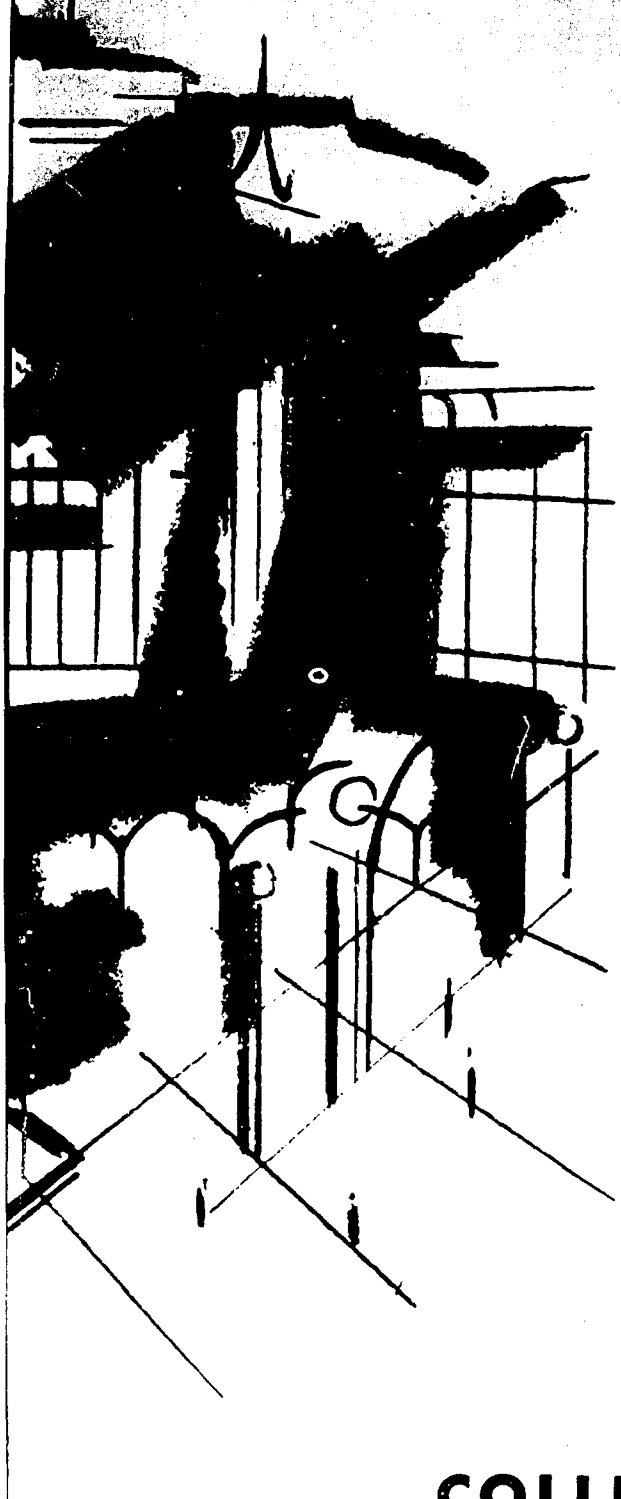

COLLAPSING SPHERICAL SHOCK WAVES IN WATER

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Robert L. Welsh

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Robert L. Welsh*

FACULTY INVESTIGATOR:
M. HOLT, PROFESSOR OF AERONAUTICAL SCIENCES

*On leave from the University of Strathclyde.

Division of Aeronautical Sciences
University of California
Berkeley, California 94720

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INTRODUCTION

The work presented here is related to various problems concerned with the propagation of shock waves, mainly imploding spherical and cylindrical shock waves, in water and (in part) in gases.

In Section 1 the relevant equation of state for water is presented and the following section deals with the consequences of this equation on shock wave transitions.

Section 2 is concerned with the self-similar solution valid in the final stages of collapse, where the shock may be considered to be infinitely strong so that the pressure ahead, and more important, a pressure term in the equation of state may be neglected. Numerical results are given for the exponent in the power law for the shock speed. In the following section a perturbation on this solution, linearized in terms of the previously neglected pressure terms, is evaluated. In effect this extends the range of validity of the self-similar solution further away from the collapse point or axis. The work of these two sections is closely related to the propagation of shocks and detonations in gases (Welsh, 1966), where a much fuller account is presented.

An approximate solution for the shock motion from the time of its initiation to its final collapse is given in Section 5, the method employed being similar to that used by previous authors (Chester, et.al.) for the motion of a shock wave in a non-uniform tube.

The final section is devoted to the exact, linearized solution for the initial motion due to the release of a spherical diaphragm separating two uniform regions at different pressures. The analysis is applicable to i) spherical or cylindrical geometry, ii) high pressure in the interior or exterior, and iii) any combination of gas and water for the two media.

1. EQUATION OF STATE FOR WATER

In the classical theory of hydrodynamics the compressibility of water is neglected, due to the relatively low order of magnitude of the pressure variations involved. However, when water is subjected to extremely high pressures, as, for instance, in an explosion, the compressibility can no longer be neglected. In such a situation the Tait equation of state is known to give an adequate representation of the compressibility (Cole, 1948). This equation relating pressure, entropy, and density can be expressed as

$$p + B(s) = B(s) \left(\frac{\rho}{\rho'} \right)^{\gamma} \tag{1.1}$$

$B(s)$ is a slowly varying function of entropy s and has the dimensions of pressure. ρ' is the (finite) value of the density for which the pressure p is zero. γ is dimensionless and approximately constant. With $B(s)$ taken to have the constant value 3.047 kilobars and γ taken as 7.15 the above equation of state gives a good approximation to the behavior of water at pressures up to 100 kilobars. Here we shall take $\gamma = 7$.

Defining the sound speed of water c by

$$c^2 = \left(\frac{\partial p}{\partial \rho} \right)_s$$

we obtain from (1.1)

$$c^2 = B(s) \frac{\rho^{\gamma-1}}{\rho_o'^{\gamma}}$$

$$= \frac{\gamma[p+B(s)]}{\rho} \tag{1.2}$$

For a perfect gas the equation of state is

$$p = e^{s/c_v} \rho^{\gamma} \tag{1.3}$$

where

$$\gamma = \text{adiabatic indices}$$

$$c_v = \text{specific heat at constant volume,}$$

which gives

$$c^2 = \left(\frac{\partial p}{\partial \rho}\right)_s = \frac{\gamma p}{\rho} \tag{1.4}$$

If we assume that the function $B(s)$ in (1.1) has the constant value B and that p is sufficiently large that we may neglect B on the left hand side of (1.1), then the equation of state for water is simply

$$p \propto \rho^\gamma$$

which is the equation of state of (1.3) for a gas when the entropy is uniform, i.e., the motion is homentropic. Also the sound speed (1.2) reduces to

$$c^2 = \frac{\gamma p}{\rho}$$

so that the motion of water at sufficiently high pressures may be treated as that of a gas with $\gamma = 7$, neglecting the effect of any variations in entropy. However, p must not be so large that (1.1) is not valid. These simplifications will be used in evaluating the self-similar solution for the final collapse of spherical shock wave.

There are evidently three distinct ranges of pressure. The first for $p \ll B$ corresponds to incompressible flow. The present work will be concerned with the remaining two, i.e., $p \sim B$ and $p \gg B$. B will be assumed constant.

We shall later require an expression for the specific internal energy of water, defined by

$$e = -\int p \, d\left(\frac{1}{\rho}\right)$$

$$= \frac{c^2}{\gamma(\gamma-1)} + \frac{B}{\rho} \qquad (1.5)$$

2. SHOCK TRANSITIONS

The propagation of a shock wave in water is governed by the mechanical laws of conservation of mass, momentum, and energy. The shock is assumed to be a discontinuity in the flow variables, viscosity and heat conduction being neglected. Also the relations between the flow variables on either side of a plane shock will be used for curved shocks, under the assumption that the width of the shock is negligible in comparison with its radius of curvature.

With the subscript o denoting the variables ahead of the normal shock, the variables behind it are given by the three conservation laws

$$\rho(U-u) = \rho_o(U-u_o) \quad ,$$

$$p + \rho(U-u)^2 = p_o + \rho_o(U-u_o)^2 \quad ,$$

$$\frac{1}{2}(U-u)^2 + \frac{p}{\rho} + \frac{c^2}{\gamma(\gamma-1)} + \frac{B}{\rho}$$

$$= \frac{1}{2}(U-u_o)^2 + \frac{p_o}{\rho_o} + \frac{c_o^2}{\gamma(\gamma-1)} + \frac{B}{\rho_o}$$

or

$$\frac{1}{2}(U-u)^2 + \frac{c^2}{\gamma-1} = \frac{1}{2}(U-u_o)^2 + \frac{c_o^2}{\gamma-1}$$

Also the second may be written

$$c^2 + \gamma(U-u)^2 = \frac{U-u}{U-u_o}(c_o^2 + \gamma U^2)$$

so that B does not appear if the equations are expressed in terms of u, c, ρ, in which case they coincide with those for a gas. We shall take $u_o = 0$.

The solution in terms of the pressure ratio $z = p/p_o$ is

$$\frac{\rho}{\rho_o} = \frac{\frac{2\gamma B}{p_o} + (\gamma-1) + (\gamma+1)z}{\frac{2\gamma B}{p_o} + (\gamma+1) + (\gamma-1)z} ,$$

$$U^2 = \frac{\gamma B}{\rho_o} + \frac{\gamma-1}{2}\frac{p_o}{\rho_o} + \frac{\gamma+1}{2}\frac{p_o}{\rho_o} z ,$$

$$u^2 = \frac{p_o}{\rho_o} \cdot \frac{(z-1)^2}{\frac{\gamma B}{p_o} + \frac{\gamma-1}{2} + \frac{\gamma+1}{2} z}$$

$$(\gamma+1)^2 c^2 = \frac{\gamma B}{\rho_o}(\gamma^2+4\gamma-1) + 4\gamma^2\frac{p_o}{\rho_o} + \gamma(\gamma^2-1)\frac{p_o}{\rho_o} z$$

$$- \frac{2\gamma^2(\gamma-1)}{\rho_o} \frac{(p_o+B)^2}{\gamma B + \frac{\gamma-1}{2} p_o + \frac{\gamma+1}{2} p_o z}$$

The special case $B = 0$ corresponds to a gas.

In terms of the Mach number $M = U/c_o$ the variables behind the shock are

$$\frac{\rho}{\rho_o} = \frac{(\gamma+1)M^2}{(\gamma-1)M^2+2} ,$$

$$u = \frac{2c_o}{\gamma+1}\left(M - \frac{1}{M}\right) ,$$

$$\frac{p}{p_o} = \frac{2\gamma M^2-\gamma+1}{\gamma+1} + \frac{2\gamma}{\gamma+1}\frac{B}{p_o}(M^2-1) ,$$

$$(\gamma+1)^2\frac{c^2}{c_o^2} = \frac{1}{M^2}[2\gamma M^2-(\gamma-1)][(\gamma-1)M^2+2] .$$

The shock Mach number M in terms of the pressure ratio is

$$M^2 = 1 + \frac{\gamma+1}{2\gamma\left(1 + \frac{B}{p_o}\right)} (z-1) \quad .$$

Since $p_o/B \doteq 1/300$, if the water is initially at atmospheric pressure, the factor multiplying $z - 1$ is approximately 5.10^3. Hence M does not differ greatly from unity unless z is of corresponding order of magnitude. In the limit as $z \to 1$, $M \to 1$ and

$$U \to c_o = \sqrt{\frac{\gamma(p_o+B)}{\rho_o}} \gg \sqrt{\frac{\gamma p_o}{\rho_o}}$$

so that a very weak shock, which is approximately sonic, travels at a much greater speed than a weak shock in a gas, but it requires a much greater change in pressure ratio to increase the speed of the shock wave in water appreciably.

Strong Shock Relations

A useful simplification of the conservation equations for a shock wave in a gas is obtained by assuming that the shock is 'strong' so that the pressure ahead may be neglected in comparison with the pressure behind. This simplification is essential in obtaining self-similar solutions involving shock waves whose speed is non-constant. In the case of water, identical strong shock relations may be obtained, but under the more severe restriction that not only is $p \gg p_o$ but also $p \gg B$. These are

$$\frac{\rho}{\rho_o} = \frac{\gamma+1}{\gamma-1}$$

$$u = \frac{2}{\gamma+1} U$$

$$c^2 = \frac{2\gamma(\gamma-1)}{(\gamma+1)^2} U^2 \quad .$$

3. SELF-SIMILAR COLLAPSING SHOCKS

The self-similar solution for the final stages of a collapsing spherical shock wave in a gas is well known (Guderley, 1942). A similarity hypothesis is obtained by using the strong shock relations, so that at any instant the flow is determined only by the radius of the shock and the initial density of the gas, apart from scaling factors.

The equations governing the symmetric motion of a perfect, inviscid, non-heat-conducting gas are

$$\left.\begin{aligned} \frac{\partial}{\partial t}(u\pm kc) + (u\pm c)\frac{\partial}{\partial R}(u\pm kc) &= \mp\frac{juc}{R} + \frac{c^2}{\gamma}\frac{\partial\varphi}{\partial R} \\ \frac{\partial\varphi}{\partial t} + u\frac{\partial\varphi}{\partial R} &= 0 \end{aligned}\right\} \qquad (3.1)$$

where $k = 2/(\gamma-1)$, $\varphi = s/[c_v(\gamma-1)]$, where s is specific entropy, and $j = 1$ for cylindrical symmetry, 2 for spherical symmetry. The appropriate equations for water are the above, neglecting entropy variations, and so are

$$\frac{\partial}{\partial t}(u\pm kc) + (u\pm c)\frac{\partial}{\partial R}(u\pm kc) = \mp\frac{juc}{R} \qquad (3.2)$$

No dimensional constants appear in these equations so that the shock speed U is dependent only on the radius coordinate λ of the shock front, the time t, and the density ρ_0. Hence,

$$U \propto \frac{\lambda}{t}$$

i.e.,

$$t \propto \lambda^{1/\alpha}$$

or

$$U \propto \lambda^{1-(1/\alpha)}$$

and, by suitable choice of length scale,

$$U = -\lambda^{1-(1/\alpha)} ,$$

$$t = -\alpha\lambda^{1/\alpha} .$$

Due to the form of the strong shock relations, the flow is self-similar with similarity variable

$$\xi = \frac{t}{\alpha R^{1/\alpha}}$$

and the dimensionless fluid velocity and sound speed r, s defined by

$$r = \frac{t}{\alpha R} u \qquad , \qquad s = \frac{t}{\alpha R} c$$

are of the form

$$r = r(\xi) \qquad , \qquad s = s(\xi) \qquad ,$$

so that the equations of motion are

$$\xi \frac{d}{d\xi}(r \pm ks) = \frac{b_\pm}{1-r\mp s}$$

where

$$b_\pm = \{1-\alpha(r\pm s)\}(r\pm ks) \mp j\alpha rs$$

We can eliminate ξ from these equations and consider r as a function of s given by

$$\frac{1}{k}\frac{dr}{ds} = \frac{(1-r+s)b_+ + (1-r-s)b_-}{(1-r+s)b_+ - (1-r-s)b_-}$$

$$= \frac{1}{s} \cdot \frac{r(r-1)(\alpha r-1) + s^2[k(1-\alpha)-\alpha(j+1)r]}{(1-r)[k-\alpha(j+k+1)r] + r(1-\alpha r)-k\alpha s^2} \tag{3.3}$$

The parameter α is as yet unknown and is determined, as in the Guderley solution, by the condition that the flow is regular on the negative characteristic which reaches the center of symmetry at the same instant, $t = 0$, as the shock wave. On this line $r = 1+s$ and the regularity condition is simply $b_- = 0$ or $r = 1+s$. The solution has to satisfy this regularity condition, at $s = s_o$ say, and the conservation equations at the shock, where

$$r = \frac{2}{\gamma+1}$$

$$s = -\frac{\sqrt{2\gamma(\gamma-1)}}{\gamma+1}$$

If we seek a regular expansion for $r(s)$ about $s = s_o$, say

$$r = r_o + r_1(s-s_o) + \ldots + r_n(s-s_o)^n + \ldots$$

then r_o, r_1 are the solutions of quadratic equations. The roots for r_o are real in some range $\alpha_R < \alpha < 1$ say, and the roots for r_1 will also be real unless this singular point (r_o, s_o) in the r-s plane is a spiral. Thus, there are in general four solutions which are regular on the limiting negative characteristic. The selection of the solution is decided partly by trial and error by computation and partly by the behavior of the integral curves of the equation (3.3) to be discussed later.

In order to integrate the equation (3.3) a similar procedure to that used previously (Welsh, 1966) was adopted. In this the regular series solution is developed iteratively, without evaluation of the actual coefficients, from the l. n. c. $s = s_o$ to the shock $s = -E$, and the discrepancy there noted. The actual solution and the appropriate value of α may then be evaluated by trial and error.

Exact results were found by this method and are given in Table I for twelve cases, $j = 1$ and $j = 2$ and $\gamma = 6/5, 7/5, 5/3, 3, 5, 7$.

In the case of a gas the extremely simple, approximate evaluation of α given by Whitham (1958) gives remarkable accuracy. The method is applicable here and consists of applying the characteristic condition to be satisfied by the variables on the l. n. c. to the values at the shock. The characteristic condition is

$$d(u-kc) = \frac{juc}{R}\,dt \qquad \text{on} \qquad \frac{dR}{dt} = u-c$$

and on the shock

$$u = -\frac{2}{\gamma+1}R^{1-(1/\alpha)}$$

$$c = E\,R^{1-(1/\alpha)}$$

and substitution of these into the characteristic condition gives

$$1-\frac{1}{\alpha} = \frac{-jDE}{(D+E)(D+kE)} \quad , \qquad D = \frac{2}{\gamma+1} \tag{3.4}$$

which is tabulated as α_w along with the exact results. The relation between α_w and the exact value is apparently more systematic than for the case of a gas, in which entropy variations occur. In all cases α_w lies between α_R and the exact values which are themselves never far apart, and the accuracy increases with decreasing γ. For $\gamma = 1$, $\alpha_w = 1$ as is the exact value. For this special case the shock is in fact a characteristic. Also, the internal energy is infinite, causing the collapse to be uniform.

The Integral Curves and the Reflected Shock

The equations (3.3) apply to the homentropic self-similar flow of a gas or the self- similar flow of water, under the present assumptions. They are identical to those used by Hunter (1960,1963) for the collapse of an empty spherical cavity in water, apart from his use of the dimensionless square of the sound speed. We shall now consider the behavior of the integral curves of (3.3).

Due to the similarity of the flow the variables are functions of ξ only. The physical R-t plane may be considered to be made of lines (parabolas) ξ = constant. The shock is $\xi = -1$ and $\xi < -1$ is the stationary region ahead of the shock. The speed of a line

$$\xi = \frac{t}{\alpha R^{1/\alpha}} = \text{constant}$$

is

$$\dot{R} = \frac{\alpha R}{t} .$$

Hence the nondimensionalization of the variables u,c to r,s is with respect to the speed of the line ξ = constant, and such a path in the R-t plane gives rise to a point in the r-s plane. The shock front transforms into the point

$$r = \frac{2}{\gamma+1} , \qquad s = \frac{\sqrt{2\gamma(\gamma-1)}}{\gamma+1}$$

Negative values of s correspond to negative values of t, i.e., to the imploding phase, and positive values of s describe the expanding phase. The positive and negative characteristics of the flow each correspond to a point on the lines $r = 1 \mp s$, respectively, on which $dr/d\xi$, $ds/d\xi$ are singular in general. Any physical solution must have ξ increasing throughout the associated integral curve in the r-s plane. In general the direction of increasing time on the integral curves in the r-s plane changes on crossing $r = 1 \pm s$. In other words, a physical solution cannot in general cross these lines. However, the appearance of singular points of the equation (3.3) for dr/ds on $r = 1 \pm s$ provide the means of crossing these lines. In effect, the singular points cause another change of direction, cancelling that appearing already. The regularity condition mentioned previously is in fact the condition for a regular solution through one of these singular points. For $\alpha < \alpha_R$ these points do not appear. for $\alpha = \alpha_R$ they coincide, and for $\alpha > \alpha_R$ they are real and distinct. If r_1 is real then there is the possibility of a regular solution through the singular point, which is either a node or a saddle. If r_1 is imaginary then the point must be a spiral.

The finite singular points of (3.3) are as follows:

$$P_1(0,1),\quad P_2(s_{o+}, 1+s_{o+}),\quad P_3(s_{o-}, 1+s_{o-}),\quad P_4(0,0)$$

$$P_4(0,0),\quad P_5\left(s, \frac{k}{\alpha(j+k+1)}\right),\quad P_6\left(0, \frac{1}{\alpha}\right)$$

and three other points which are the mirror images of P_2, P_3, P_5 in the r-axis. Both $s_{o\pm} < 0$, and S is the negative root of

$$S^2 = \frac{r(1-\alpha r)}{k\alpha}, \qquad r = \frac{k}{\alpha(j+k+1)}$$

P_4 is always a degenerate note and P_1 is a saddle point with asymptotes in the direction of the r,s axes. P_6 is always a node and is degenerate if $k = j+1$ (i.e., $\gamma = 5/3$ spherical, $\gamma = 2$ cylindrical). For $k > j+1$ the major axis, i.e., the direction of all the integral curves except for one at right angles, is in the r-direction and for $k < j+1$ in the s-direction. For P_5 there are two possibilities. When it is above $r = 1+s$ it is a node and a saddle when below, the transition occurring through its merging with P_2 or P_3 on $r = 1+s$. Except in the special case $k = j+1$ the points P_2, P_3 separate into a node and saddle or saddle and node, respectively, for α slightly greater than α_R. When they coincide with $\alpha = \alpha_R$ they form a confluence of a node and saddle. This type of singular point is not a simple one. In investigating the behavior of the curves near such a point it is necessary to include second order terms in the numerator and denominator as the discriminant of the coefficients of the first order terms vanishes, and to first order the solution obtained is a straight line. For such a second order singular point the significant variations occur on parabolas. For a first order node or saddle the field of curves may be determined by finding the asymptotes and the lines on which the curves have zero and infinite slope. For a second order singular point the latter two lines and the asymptote between them are parabolas and the relative positions of the curves are the same on either side of the point (in the first order case the positions are reversed) resulting in a naddle. This situation is exhibited graphically in Fig. 1. In the special case $k = j+1$ there is a triple coincidence of P_2, P_3 and P_5 when $\alpha = \alpha_R$, resulting in a third order singular point, so that the significant variations take place along cubic curves. The position of these differs on either side of the point and the situation is geometrically similar to the first order case, except for the scale involved. Such a point may be either a third order node or saddle. Evidently higher order singular points will be geometrically similar to the first order type of singular point (node or saddle) or the second order type (naddle), depending on whether the order is odd or even.

The values of α for which these transitions just discussed occur are tabulated in Table I. For $k < j+1$ the point P_3, which is a node when it first appears, changes to a spiral, in which case no regular solution through it is possible. However, the actual solution was found to pass through P_2 in all cases. Through a first order node or saddle there are two regular solutions. In the case of a saddle these are the asymptotes, which are the only two solutions passing through the actual point. For a node one of the two is the minor axis, i.e., the one solution not having the same slope as all the others at the point, but the other is one of the remaining curves and hence is less distinguishable. In all cases the actual solution was through the major axis of P_2, and this was found to be the only possible solution. No transition from P_2 to P_3 occurs as it does in the case of a gas (Welsh, 1966). Also the argument given in that publication that no uniform collapse $(\alpha = 1)$ is possible applies here to the case of water, since the flow behind a uniform shock is homentropic and the equations of motion for gas and water are identical. This contrasts with the situation found by Hunter (1960,1963) for cavitation in water. An accelerating collapse exists in a certain range of γ, a uniform one in a different range, the latter being singular at the cavity front for certain discrete values of γ.

The appropriate integral curve representing the actual solution in a given physical solution starts at the shock point $(D,-E)$, crosses $r = 1+s$ at P_2, and then runs into $P_4(0,0)$, with time increasing throughout this path. P_4 corresponds to the line $t = 0$ in the physical plane, and the solution must continue beyond P_4. The integral curves are symmetric about $r = 0$, except for a change in direction of increasing time due to the change in sign of s. Thus the integral curve passes through P_4 smoothly, the points of the curve beyond P_4 corresponding to lines $\xi > 0$, i.e., expanding lines in the flow behind the shock. However, the curve heads towards the singular line $r = 1-s$, corresponding to a positive characteristic in the flow, which indicates the presence of a reflected shock wave traveling along $\xi = \text{constant} > 0$. The solution is represented by a portion of this curve in the region $s > 0$, $r < 1-s$, but jumps to some other curve above $r = 1-s$. A condition on the flow behind this reflected shock is that the fluid velocity be zero at the center, i.e.,

$u = 0$ on $R = 0$. Hence r is finite here, but since c is finite, s is infinite. It will now be seen that this information is sufficient to determine the integral curve which satisfies these conditions at $R = 0$, by examining the singular points of (3.3) at infinity.

Here we are interested in the region $s > 0$. There are three singular points on $s = +\infty$ at $r = \pm\infty, \dfrac{k(1-\alpha)}{\alpha(j+1)}$. The first two are nodes (half-nodes) and the one between them is a saddle (half-saddle), so that all curves, save one, approach either $r = \pm\infty$ as $s \to \infty$. The asymptote through the saddle point is the only curve which has a finite value of r for $s = +\infty$ and so represents the flow behind the reflected shock. The jump from this curve to the one describing the flow ahead of the reflected shock is determined by the conservation equations. In this way the value of ξ on the reflected shock can be evaluated. This has not been done here.

This type of reflection occurs when the shock speed is singular and its curvature is infinite. If there is a symmetrically placed solid spherical or cylindrical boundary causing the shock to be reflected before its final collapse then this type of reflection is plane in nature, provided the radius of the boundary is large in comparison with the shock width.

4. THE PERTURBATION OF THE SELF-SIMILAR SOLUTION

The self-similar solution is valid for $p \gg B$, which is fairly restrictive, and presumably means that it is valid for a collapsing shock wave in water for a much smaller neighborhood of the collapse point than in the case of a gas. Here we shall consider the perturbation on the self-similar solution due to taking B into account to first order. The perturbations are analogous to those due to the counter-pressure ahead of a collapsing shock wave in a gas (Welsh, 1966), the perturbations being of relative order $R^{-2+(2/\alpha)}$.

The form of the solution is now

$$u = \frac{r(\xi)}{\xi} R^{1-(1/\alpha)} + \frac{\bar{r}(\xi)}{\xi} R^{-1+(1/\alpha)}$$

$$c = \frac{s(\xi)}{\xi} R^{1-(1/\alpha)} + \frac{\bar{s}(\xi)}{\xi} R^{-1+(1/\alpha)}$$

and the equations governing these perturbations are

$$\left.\begin{aligned} \xi(1-r\mp s)\frac{d}{d\xi}(r\pm ks) &= b_\pm \\ \xi(1-r\mp s)\frac{d}{d\xi}(\bar{r}\pm k\bar{s}) &= a_\pm \end{aligned}\right\}$$

where

$$a_\pm = \mp j\alpha(r\bar{s}+r\bar{s}) + (1-r\mp s)(\bar{r}\pm k\bar{s}) \mp (1-\alpha)(k-1)(r\bar{s}-r\bar{s})$$

$$+ \frac{\bar{r}\pm\bar{s}}{1-r\mp s} b_\pm - (\bar{r}\pm\bar{s})(r\pm ks) \quad ,$$

and ξ may be eliminated as previously by considering r, $\bar{r}$, $\bar{s}$ to be functions of s

$$\left.\begin{aligned} (1-r-s)b_-(r'+k) &= (1-r+s)b_+(r'-k) \\ a_+(r'+k) &= b_+(\bar{r}'+k) \\ a_-(r'-k) &= b_-(\bar{r}'-k) \end{aligned}\right\} \qquad (4.1)$$

where $' \equiv d/ds$

As is the case in the basic self-similar solution the boundary conditions are obtained from the regularity condition at $s = s_o$ and the conservation equations at $s = -E$. Since the displacement of these lines from their positions in the self-similar flow gives rise to terms of the same order as the perturbations, it is necessary to take into account these displacements when applying boundary conditions on these lines.

In the basic flow the lines are

$$\xi = -1 \qquad , \qquad \xi = \xi_1$$

and in the perturbed flow

$$\xi = -1(1 + \beta R^{-2+(2/\alpha)}) \qquad , \qquad \xi = \xi_1(1 + \delta R^{-2+(2/\alpha)})$$

We are mainly interested in the value of β.

The boundary conditions at the shock are obtained by solving the conservation equations with c_o taken into account to first order. With the problem expressed in terms of u and c, β does not appear explicitly and we have

$$u = D\,U - D'\,U^{-1}$$

$$-c = E\,U + E'\,U^{-1}$$

where

$$E' = \frac{6\gamma-\gamma^2-1}{2(\gamma+1)\sqrt{2\gamma(\gamma-1)}} \cdot c_o^2 \quad , \quad D' = \frac{2}{\gamma+1}\,c_o^2$$

$$\text{and} \qquad U = -R^{1-(1/\alpha)}\,(1 + \beta R^{-2+(2/\alpha)})$$

Hence at $u = -E$ we have

$$\left.\begin{aligned} \bar{r}(-E) &= \frac{\beta D}{3-2\alpha}\left[j\alpha\gamma D - \frac{2(1-\alpha)(2\gamma-1)}{\gamma-1} + 3 - 2\alpha \right] - D' \\ \bar{s}(-E) &= -\frac{\beta E}{3-2\alpha}\left[j\alpha\frac{\gamma-1}{\gamma+1} + \alpha \right] - E' \end{aligned}\right\} \qquad (4.2)$$

The value of c_o^2 is given by

$$c_o^2 = \frac{\gamma(p_o+B)}{\rho_o}$$

and in general

$$c_o^2 \doteq \frac{\gamma B}{\rho_o} \quad .$$

In the calculations the value unity will be used for c_o^2, the solution in a given case being given by the appropriate multiple of this solution.

The boundary conditions at $s = s_o$, due to the solution being regular there, are obtained as follows. This line is a characteristic so that, on it

$$\frac{dR}{dt} = u - c$$

and hence

$$\frac{1}{\xi_1} R^{1-(1/\alpha)} - \frac{3-2\alpha}{\xi_1} \delta R^{-1+(1/\alpha)} = \frac{r_o-s_o}{\xi_1} R^{1-(1/\alpha)}$$
$$+ \left\{ \delta \frac{d}{d\xi} \left(\frac{r}{\xi} - \frac{s}{\xi} \right)_{\xi_1} + \frac{\bar{r}_1-\bar{s}_1}{\xi_1} \right\} R^{-1+(1/\alpha)}$$

so that

$$\left.\begin{aligned} r_o - s_o &= 1 \\ \bar{r}_o - \bar{s}_o &= \left\{ 1 + \frac{b_{o+}(r_1-1)}{s_o(r_1+k)} \right\} \delta - (3-2\alpha)\,\delta \end{aligned}\right\} \qquad (4.3)$$

The regularity condition is obtainable either by application of the characteristic condition on $\xi = \xi_1$ or else by observing that the first of Eq. (4.1) implies

$$b_{o-} = 0 \quad , \qquad \text{where } o \text{ denotes the value at } s = s_o$$

and the third implies

$$a_{o-} = 0$$

which is

$$j\alpha(r_o\bar{s}_o+\bar{r}_o s_o) + (1-\alpha)(k-1)(r_o\bar{s}_o-\bar{r}_o s_o)$$

$$- (\bar{r}_o-\bar{s}_o)\,\frac{(r_1-k)b_{o+}}{2s_o(r_1+k)} - (\bar{r}_o-\bar{s}_o)(r_o-ks_o) = 0 \qquad (4.4)$$

Hence $\bar{r}_o$, $\bar{s}_o$ are known in terms of δ.

Let us take any trial value of δ, $\delta^{(o)}$ say, and suppose that the values of $\bar{r}$, $\bar{s}$ at $s = -E$ thus obtained are denoted by $\bar{r}_H^{(o)}$, $\bar{s}_H^{(o)}$, respectively. Since the system is linear any multiple of this solution is also a solution of the differential equations. We require that a multiple X of this solution satisfies (4.3), i.e.,

$$\left.\begin{aligned} X\bar{r}_H^{(o)} &= A_1\ \beta - D' \\ X\bar{s}_H^{(o)} &= A_2\ \beta - E' \end{aligned}\right\} \qquad (4.5)$$

in which X, β are unknown. Thus we can evaluate β and also X if it is desired to tabulate the solution (this is not done here).

The application of Whitham's rule, that the flow variables at the shock, $\xi = -1$, satisfy the characteristic condition at $\xi = \xi_1$, gives a simple algebraic formula for the approximate evaluation of β

$$\beta_w = \frac{D'E-E'D}{2DE} + \frac{(1-k)(D'E+DE')}{2(D+E)(D+kE)}$$

However, one cannot expect this approximation to have much accuracy.

The evaluation of β by this method is extremely simple due to the fact that it is entirely algebraic. The scheme applied to evaluate β exactly (Welsh, 1966) could also be applied here. However, this involves the exact integration of the system (4.1). Instead of doing so it is proposed to apply an approximate algebraic method, the accuracy of which may be gauged by comparing the results of this method to the gas case with the exact values. In theory, one could develop the exact regular solution of (4.1) as a power series about $s = s_o$, although the algebra involved becomes impossible after the second terms, whose values are needed for the exact method. We shall make use of the fact that the range of integration is quite small, of order 0.2, and evaluate the series solution only as far as the second term, of order $s = s_o$. In other words, we shall evaluate $\bar{r}_1$, $\bar{s}_1$ for $s = s^{(o)}$ and replace $\bar{r}_H^{(o)}$, $\bar{s}_H^{(o)}$ on the left hand side of (4.5) by

$$\left.\begin{aligned} \bar{r}_H^{(o)} &= \bar{r}_o^{(o)} + \bar{r}_1^{(o)}(s_H - s_o) \\ \bar{s}_H^{(o)} &= \bar{s}_o^{(o)} + \bar{s}_1^{(o)}(s_H - s_o) \end{aligned}\right\}$$

and denote the value of β so obtained by β_1 (the first order solution). The less accurate, zero order solution β_o will also be found by truncating the series after a single term so that $\bar{r}_H^{(o)}$, $\bar{s}_H^{(o)}$ are replaced by $\bar{r}_o^{(o)}$, $\bar{s}_o^{(o)}$, respectively.

The values of β_o, β_1, β_w are given for $\gamma = 7$, $j = 1,2$ in Table II along with the entropy varying case for $\gamma = 3$ (Welsh, 1966).

5. THE APPROXIMATE SOLUTION TO THE COMPLETE MOTION

The work of this section arises from the work of Chester (1954), Chisnell (1957), and Whitham (1958), on plane shock waves propagating along a tube of varying cross-sectional area A. Evidently $A \propto R^j$, $j = 1,2$, corresponds to wedge and cone-shaped tubes and hence to complete cylindrical and spherical shock fronts. The method employed by these authors, which is in effect to apply Whitham's rule taking complete account of the state ahead of the shock, and so is not restricted to strong shocks, gives good agreement with the numerical solution of the complete system of partial differential equations obtained by Payne (1957).

The characteristic condition is

$$du - kdc = -\frac{1}{\gamma} c d\varphi + \frac{juc}{u-c} \frac{dR}{R}$$

where the φ term is omitted for water,

$$u = \frac{2c_o}{\gamma+1} \left(M - \frac{1}{M}\right)$$

$$c = -\frac{c_o}{(\gamma+1)M} \sqrt{\{2\gamma M^2-(\gamma-1)\}\{(\gamma-1)M^2+2\}}, \quad \text{since } M = \frac{U}{c_o} < 0$$

This results in the formula

$$\frac{jdR}{R} + \frac{2M}{M^2-1} \frac{dM}{K(M)} = 0$$

for the shock propagation. If we denote $K(M)$ for a gas by $K^{(o)}(M)$, then Chester's formula is

$$\frac{2}{K^{(o)}(M)} = \left\{1 + \frac{2(M^2-1)}{\sqrt{xy}}\right\} \left\{1 + \frac{1}{M^2} + 2\sqrt{x/y}\right\} \qquad (5.1)$$

where $x = (\gamma-1)M^2+2$

$y = 2\gamma M^2-\gamma+1$

and the formula for water in which φ is neglected is

$$\frac{2}{K^{(1)}(M)} = \left\{1 + \frac{2(M^2-1)}{\sqrt{xy}}\right\}\left\{1 + \frac{1}{M^2} + \frac{2}{M^2}\,\frac{\gamma M^4+1}{\sqrt{xy}}\right\} \qquad (5.2)$$

In the limit $M \to 1$, for weak shocks, $K \to 1/2$ in both cases for all γ, so that the integral for M is

$$M - 1 \propto R^{-j/2} \quad ,$$

the law for acoustic propagation.

In the limit as $M \to \infty$ the integral

$$M \propto R^{-(j/2)K_\infty}$$

so that $K_\infty = (1/\alpha_w)-1$, spherical case.

Graphs of $K^{(o)}(M)$ and $K^{(1)}(M)$ are given in Fig. 2 for several cases, the one most relevant to the present work being $K^{(1)}(M)$ for $\gamma = 7$. These results give an idea of the effect of neglecting entropy variation, which is substantial for $\gamma = 1.4$, but decreases as γ increases, and is not nearly so great for $\gamma = 7$. For a given γ, $K^{(1)}(M) < K^{(o)}(M)$ for all M, so that the entropy being neglected causes the shock to accelerate more rapidly. K is mainly monotonic, increasing for large γ and decreasing for small γ. For $\gamma = 2$, $K_\infty^{(1)} = 1/2$ and $K^{(1)}(M)$ varies very little, so that

$$M^2 - 1 \propto R^{-j/2}$$

is a very accurate approximation for all M.

The results also relate to the propagation of a shock wave in a tube of varying cross-section, the general area charge dA/A replacing the term jdR/R.

6. THE SPHERICAL AND CYLINDRICAL SHOCK TUBE

The self-similar solution for the final stages of the collapse of an imploding shock wave is independent of the means by which the shock is initiated, except for scaling factors. This also applies to perturbations to the self-similar solution obtained by taking account of the pressure ahead of the shock.

Here we shall consider the initial propagation of the shock wave with the initial state of a spherical pressure discontinuity separating two uniform regions. The appearance of the initial radius of the sphere r_o as a parameter means that the flow is non-similar. However, in the limit as the instant of release is approached, the flow is independent of r_o and is given by the classical solution for the one-dimensional shock tube, caused by the release of the plane diaphragm separating two uniform media at different pressures. This solution is self-similar. The only parameters have the dimensions of pressure and density (r_o neglected) so that the similarity variable is r/t and the flow is uniformly expanding. In this solution the pressure ahead of the shock is taken into account exactly (for the final collapse the similarity variable is r/t^{α}, $0 < \alpha < 1$ and similarity demands that the pressure ahead of the shock be neglected).

As the shock propagates initially disturbances will arise in the flow due to the departure from plane geometry, and these will cause an imploding shock to accelerate and an expanding one to decelerate. These disturbances will be treated as a perturbation on the basic, one-dimensional, self-similar solution, to obtain the initial acceleration of the shock wave. The analysis will be applicable to any combination of gas and water for the two media, and also to either the sphere being at a lower pressure than its exterior or the reverse situation. Since it is known that the final collapse rate is singular, a series obtained as a perturbation will certainly not be valid up to the moment of collapse, and can only describe the initial motion (i.e., for small values of $[r-r_o]/r_o$).

The representation of the one-dimensional shock tube flow, with $r = r_o$ as the diaphragm position is shown in Fig. 4. For this diagram the high pressure region is $r < r_o$. The gas in the lower pressure region is compressed through a shock wave and that in the high pressure region is expanded through a simple wave. The regions 0,4 beyond the influence of these waves are in the initial stationary states. The regions 2,3 are in uniform motion with the same pressure and fluid velocity. The contact front separates the two media. The region 1 is a point-centered simple rarefaction wave.

In the one-dimensional system this flow would persist for all time in the absence of any dissipation. The disturbances on this flow due to a spherical or cylindrical geometry will start from $t = 0$ and will be small in the initial stages. These disturbances will travel along characteristics, so let us consider the overall picture of the disturbances. Consider first the fluid in the high pressure region $r < r_o$, i.e., the regions 0, 1, 2. No disturbances can propagate in the region 0, which remains uniform. Also the path of the last characteristic of the simple wave, separating 0 and 1, is not disturbed. In the expansion wave, region 1, there will be one degree of freedom for each family of characteristics along which disturbances may propagate. By a degree of freedom we mean an arbitrary constant in the general solution for the region, and these constants which arise will be determined from appropriate boundary conditions. In any region there are three families of characteristics, $c^{(\pm)}$ and $c^{(o)}$ defined by $d/dt = u \pm c$, $d/dt = u$, respectively. Thus in any region there will be at most three degrees of freedom. There can be no entropy disturbance in the simple wave, so that there is no degree of freedom associated with the $c^{(o)}$ characteristics, i.e., the particle paths. The negative characteristics are intersecting straight lines through $r = r_o$, $t = 0$. In the basic flow this point is singular as the characteristics intersecting here have different values of the characteristic variable. Since all perturbations must be zero at $t = 0$ and all these lines intersect at $t = 0$, it follows that there can be no disturbances propagating along $c^{(-)}$. Hence there is only one degree of freedom in the simple wave due to disturbances along $c^{(+)}$, and this will be determined by the condition that it is zero on the last characteristic of the simple wave.

There is no entropy disturbance in region 2 but there will be disturbances on $c^{(+)}$ and $c^{(-)}$, the former coming from the simple wave and the latter from the shock. The solution in this region has to be matched with that on the first characteristic of the simple wave. This condition determines the constant associated with $c^{(+)}$ in 2 to that of $c^{(+)}$ in 1 as the disturbances propagating along $c^{(-)}$ in region 2 do not enter into this matching.

Thus the motion of the gas to the left of the contact front can be described except for the degree of freedom associated with $c^{(-)}$ in region 2, which will be determined by matching this solution with that for the gas initially in $r > r_o$, along the contact front.

The region 4 ahead of the shock is unperturbed. However, the path of the shock itself will be altered, causing disturbances along all three families of characteristics in the region 3 behind it. The conservation equations across the shock serve to determine the boundary values of the solution in terms of the change in shock speed, so that the flow in this region is expressible in terms of one unknown, the change in shock speed.

There are two conditions to be satisfied at the contact front, the continuity of pressure and fluid velocity. Since the boundary values to the left and right of the front each contain one unknown, these two conditions will determine the flow completely.

The characteristics along which disturbances propagate are sketched in Fig. 3. The problem is greatly simplified if the disturbances propagating along $c^{(+)}$ in region 3 are neglected, as the propagation of the shock wave is then determined simply by examination of the boundary values at the shock, without reference to the motion behind the shock. This situation would arise if there were no simple wave and the regions 0,1 were identical to 2. An extensive region of uniform flow behind the shock cannot arise in an entirely spherical or cylindrical geometry. However, it describes a uniform plane shock wave traveling in a one-dimensional tube and meeting an appropriately varying area change. This is the case studied by Chester (1954), Chisnell (1957), and Gundersen (1958). For a very weak shock, which is nearly sonic, the disturbances reaching it along $c^{(+)}$ take a long time to do so and hence their effect will be increasingly small for increasingly weak shocks. The simplest derivation of the formula for the shock propagation neglecting these disturbances is that of Whitham (1958), and it will be shown that neglecting them in the present analysis reduces to his formula. The release of a pressurized sphere in air has been studied by Friedman (1961), who uses Whitham's rule for the propagation of the main shock wave.

Equations of Motion

The equations of motion for the spherically or cylindrically symmetric flow, in the absence of dissipation due to heat conduction, are those given previously in Section 3. Defining the characteristic variables

$$\alpha^{\pm} = \frac{1}{2} u \pm \frac{1}{\gamma-1} c$$

these are

$$\left.\begin{aligned} \alpha_t^{\pm} + (u \pm c)\alpha_r^{\pm} &= \mp \frac{j}{2r} uc + \frac{1}{2\gamma} c^2 \varphi_r \\ \varphi_t + u\varphi_r &= 0 \end{aligned}\right\} \qquad (6.1)$$

The basic self-similar flow is given by the above equations with the geometry term involving j omitted, the variables being functions of $(r-r_o)/t$ only. The perturbations to be evaluated here are essentially due to the effect of this term.

The basic solution is of the form

$$\alpha_o^{\pm} = \alpha_o^{\pm}(\lambda)$$

$$\varphi_o = \varphi_o(\lambda) \qquad , \qquad \lambda = \frac{r-r_o}{t}$$

and also U_o = constant. Let us expand about this solution in the form

$$\alpha^{\pm} = \alpha_o^{\alpha}(\lambda) + \epsilon\alpha_1^{\pm}(\lambda) + \epsilon^2\alpha_2^{+}(\lambda) + \dots , \qquad (6.2)$$

$$\text{where } \epsilon = \frac{r-r_o}{r_o}$$

We shall be concerned with only the first order corrections, i.e., those with suffix 1. On substituting the variable, expanded in this form, into the equations of motion and equating like powers of λ, we obtain the equations governing the basic flow

$$\left.\begin{aligned} (\lambda-u_o)\varphi_o' &= 0 \\ (u_o \pm c_o - \lambda)\alpha_o^{\pm\prime} &= \frac{c_o^2}{2\gamma} \varphi_o' \end{aligned}\right\} \qquad (6.3)$$

and the equations for the perturbations

$$\left.\begin{aligned}&\lambda(u_o-\lambda)\varphi_1' + u_o\varphi_1 + \lambda u_1\varphi_o' = 0\\&\lambda(u_o\pm c_o-\lambda)\alpha_1^{\pm'} + \lambda(u_1\pm c_1)\alpha_o^{\pm'} + (u_o\pm c_o)\alpha_1^{\pm}\\&\qquad = \mp\frac{1}{2}u_oc_o + \frac{c_o}{2\gamma}\left\{2\lambda c_1\varphi_o' + c_o(\lambda\varphi_1'+\varphi_1)\right\}\end{aligned}\right\} \quad (6.4)$$

The simplicity of the equations (6.3) is due to the combination of self-similarity and characteristic propagation. The first is the statement φ_o is constant except for a possible discontinuity on $\lambda = u_o$, i.e., the particle path through the point of initiation in the r-t plane. Thus the right hand side of the second is zero, and this becomes

$$(u_o\pm c_o-\lambda)\alpha_o^{\pm'} = 0$$

which states that $\alpha_o^{\pm}$ is constant except possibly on an appropriate characteristic through the point of initiation. Here, in the case of the point-centered simple wave, it is possible for there to be an intersecting family of such characteristics, giving a region in which the first factor equated to zero is the appropriate solution of this equation. For all other regions, which are uniform, $\alpha_o^{\pm}$ = constant is the solution. The characteristics of the system (6.4) are evidently the three characteristics of the flow.

The Simple Wave

Here we shall solve the system (6.3) for the familiar solution for a point-centered simple wave and hence the system (6.4) for the perturbations. In these equations a double sign $\pm$ was used for simplicity, the upper sign referring to the $c^{(+)}$ equation and the lower to the $c^{(-)}$. In order to develop the two cases of high pressure inside and outside the sphere simultaneously a similar notation will be used. The former will be referred to as case +, and will be given by the upper signs in the equations; and the later, case -, will be given by the lower signs. Thus the two signs refer here to different physical situations, whereas, as used previously, they refer to different equations describing the same physical situation. The basic simple wave is

$$\left.\begin{aligned} u_o \mp c_o - \lambda &= 0 \\ \alpha_o^{\pm} &= \alpha_{oR}^{\pm} \\ \varphi_o &= \text{constant} \end{aligned}\right\} \qquad (6.5)$$

The solution for φ_1 is

$$\varphi_1 = \frac{A}{\lambda}\{2c_{or} \mp (\gamma-1)\lambda\}^{\frac{\gamma+1}{\gamma-1}}, \qquad \begin{aligned} A &= \text{arbitrary constant} \\ c_{oR} &= c_o \text{ in region 0} \end{aligned}$$

and the equations for α_1^+ are

$$\begin{bmatrix} 2\lambda\{2c_{oR}-(\gamma-1)\lambda\} & 0 & 4c_{oR}+(3-\gamma)\lambda \\ 0 & 2\lambda & \lambda(\gamma+1) \end{bmatrix} \begin{bmatrix} \alpha_1^{+'} \\ u_1+c_1 \\ \alpha_1^+ \end{bmatrix} \qquad (6.6)$$

$$= -\frac{1}{\gamma+1}(\lambda\pm c_{oR})\{2c_{oR}\mp(\gamma-1)\lambda\} \mp \frac{A}{2\gamma}\{2c_{oR}\mp(\gamma-1)\lambda\}^{\frac{2\gamma}{\gamma-1}}$$

and for α^-

$$\begin{bmatrix} 0 & 2\lambda & (\gamma+1)\lambda \\ -2\lambda\{2c_{oR}+(\gamma-1)\lambda\} & 0 & (3-\gamma)\lambda-4c_{oR} \end{bmatrix} \begin{bmatrix} \alpha_1^{-'} \\ u_1-c_1 \\ \alpha_1^- \end{bmatrix} \qquad (6.7)$$

$$= \frac{1}{\gamma+1}(\lambda\pm c_{oR})\{2c_{oR}\mp(\gamma-1)\lambda\} \mp \frac{A}{2\gamma}\{2c_{oR}\mp(\gamma-1)\lambda\}^{\frac{2\gamma}{\gamma-1}}$$

In each pair of equations one is algebraic so that their solution has only one arbitrary constant. This is due to there being no disturgances propagating along the rectilinear, intersecting characteristics, the similarity approach ruling out such disturbances automatically.

The solution of the differential equations which appear in (6.6) and (6.7) may be expressed as

$$\lambda\alpha_1^{\pm} = \frac{-j}{(\gamma^2-1)(3\gamma-5)}(\mp) - \frac{j\,c_{oR}}{(\gamma-1)(3-\gamma)}(\mp) \pm \frac{A}{2\gamma(3\gamma-1)}(\mp)^{\frac{2\gamma}{\gamma-1}} + K_1(\mp)^{\frac{\gamma+1}{2(\gamma-1)}} \tag{6.8}$$

where $(\mp) = (2c_{oR} \mp (\gamma-1)\lambda)$, $\gamma \neq 3, 5/3$,

and $K_1^{\pm}$ are arbitrary constants.

The algebraic equations may be combined as

$$\lambda\left[\frac{\gamma+5}{2}u_1 \mp \frac{3\gamma-1}{\gamma-1}c_1\right] = \pm\frac{j}{\gamma-1}(\lambda \pm c_{oR})(\mp) \tag{6.9}$$

Since the perturbed flow is homentropic A may be set equal to zero in (6.8). The flow is continuous across the last characteristic of the simple wave separating the regions 0 and 1, i.e., $\lambda = \mp c_{oR}$, so that u_1, c_1 must be zero on this line. The equations (6.8), (6.9) for u_1, c_1 are singular only in the case $\gamma = 1$. The right hand side of (6.9) is zero on $\lambda = \mp c_{oR}$, so $K^{\pm}$ is determined by equating the right hand side of (6.8) to zero. This gives

$$K_1^{\pm} = \frac{2j(\gamma+1)^{\frac{\gamma-3}{2(\gamma-1)}}\,c_{oR}^{\frac{3\gamma-5}{2(\gamma-1)}}}{(3\gamma-5)(3-\gamma)} \tag{6.10}$$

The solution in the simple wave is now determined completely by (6.8), (6.9), with $A = 0$ and $K^{\pm}$ given by (6.10).

In the basic simple wave the first characteristic is the line on which the value of the fluid velocity is the same as that behind the shock, u_s, say. On this line

$$u_o = u_s$$

$$c_o = c_{oR} \mp \frac{\gamma-1}{2} u_s$$

$$\lambda = \frac{\gamma+1}{2} u_s \mp c_{oR}$$

The Uniform Regions 2 and 3

In the case of a uniform homentropic basic flow the equations (6.4) governing the perturbations are

$$\lambda(u_o-\lambda)\varphi_1' + u_o\varphi_1 = 0$$

$$\lambda(u_o \pm c_o-\lambda)\alpha_1^{\alpha'} + (u_o \pm c_o)\alpha_1^{\pm} = \mp \frac{j}{2} u_o c_o + \frac{c_o^2}{2\gamma} (\lambda\varphi_1'+\varphi_1)$$

where u_o, c_o are constant.

The solution of this system is

$$\left.\begin{aligned} \varphi_1 &= A \frac{\lambda-u_o}{\lambda} \\ \lambda\alpha_1^{\pm} &= \mp \frac{j}{2} u_o c_o + \frac{Ac_o^2}{2\gamma} + K'^{\pm}(u_o \pm c_o-\lambda) \end{aligned}\right\} \qquad (6.11)$$

The last term is constant on $u_o \pm c_o-\lambda = \lambda$, i.e., on $c^{(\pm)}$, but the other terms are not associated with characteristic propagation as written. The entropy term is associated with $c^{(o)}$ so we can write it as

$$\frac{Ac_o^2}{2\gamma} = \pm \frac{Ac_o}{2\gamma} (\lambda-u_o) \pm \frac{Ac_o}{2\gamma} (u_o \pm c_o-\lambda)$$

and the first term may be written

$$\frac{j}{2} u_o c_o = \frac{j}{2} \frac{u_o c_o \lambda}{u_o \pm c_o} + \frac{j}{2} u_o c_o \frac{u_o \pm c_o-\lambda}{u_o \pm c_o}$$

so that (6.11) may be expressed in the form

$$\left.\begin{aligned} \varphi_1 &= A\,\frac{\lambda-u_o}{\lambda} \\ \lambda\alpha_1^{\pm} &= \mp\frac{1}{2}\,\frac{u_o c_o \lambda}{u_o \pm c_o} + \frac{Ac_o}{2\gamma}\,(\lambda-u_o) + K^{\pm}(u_o \pm c_o - \lambda) \end{aligned}\right\} \qquad (6.12)$$

where in the last equation the terms are respectively the geometry modification, the entropy disturbance along $c^{(o)}$, and the disturbance traveling along $c^{(\pm)}$. By neglecting the disturbances reaching the shock from behind, i.e., by setting the appropriate K zero, Whitham's formula is obtained, A retained for a gas and omitted for water.

The particles in the region 2 between the simple wave and the contact front do not pass through the shock, so that $A = 0$ in region 2, and

$$\lambda\alpha_1^{\pm} = \mp\frac{1}{2}\,\frac{u_o c_o \lambda}{u_o \pm c_o} + K_2^{\pm}(u_o \pm c_o - \lambda)$$

where $K_2^{\pm}$ are both non-zero.

This flow has to match identically with the perturbed flow in the simple wave on the first characteristic of the simple wave separating the two regions, which determines one of $K_2^{\pm}$. The remaining K_2 will be determined by the conditions at the contact front. Here

$$\lambda = u_s$$

so that

$$\alpha_1^{\pm} = \mp\frac{1}{2}\,\frac{u_o c_o}{u_o \pm c_o} \pm \frac{c_o}{u_o}\,K_2^{\pm} \qquad (6.13)$$

where $u_o = u_s$, $\quad c_o = c_{oR} \mp \frac{\gamma-1}{2}\,u_s$.

The solution in the region 3 between the contact front and the shock is

$$\lambda\varphi_1 = A(\lambda - u_o)$$

$$\lambda\alpha_1^{\pm} = \mp \frac{1}{2} \frac{u_o c_o \lambda}{u_o \pm c_o} \pm \frac{A c_o}{2\gamma}(\lambda - u_o) + K_3^{+}(u_o \pm c_o - \lambda) \tag{6.14}$$

If the medium in this region is gas then A is non-zero and is determined by the conservation equations at the shock. For water, in which the effect of entropy variations is neglected, A will be zero. The conservation equations at the shock, i.e., on $\lambda = U_o$, give three equations relating A, $K_3^{\pm}$, and U_1, so that the solution in this region is expressible in terms of the single unknown U_1. The boundary values at the contact front are given by (6.14) with $\lambda = u_o$. The physical situations are here distinguished by the fact the u_o, U_o are negative for the imploding shock and positive for the exploding shock (c_o being always positive).

The continuity of fluid velocity and pressure at the contact front give two equations involving the unknown K_2 and U_1. No calculations have been performed as yet.

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TABLE I

γ	j	α_R	α_W	α_{exact}	$\alpha_{cavit.}$ (1)	P_5 transition (2)	P_2 transition	P_3 transition
7	2	0.5544	0.6062	0.633291	0.5552	0.6196	0.6196 (3)	0.5642 (4)
7	1	0.7133	0.7548	0.766808		0.7489	0.7489 (3)	0.7372 (4)
5	2	0.5930	0.6294	0.651903	0.6009	0.6377	0.6377 (3)	0.6104 (4)
5	1	0.7445	0.7726	0.782191		0.7657	0.7657 (3)	0.7821 (4)
3	2	0.6667	0.6830	0.696825	0.7086	0.6830	0.6830 (3)	0.7114 (4)
3	1	0.8000	0.8116	0.817128		0.8047	0.8047 (3)	0.8803 (4)
5/3	2	0.7887	0.7911	0.794838	0.9290 (γ=1.7)	0.7887	always node	always node
5/3	1	0.8819	0.8834	0.884659		0.8828	do.	0.8828 (3)
7/5	2	0.8396	0.8405	0.841986		0.8415	do.	0.8415 (3)
7/5	1	0.9128	0.9133	0.913793		0.9163	do.	0.9163 (3)
6/5	2	0.8965	0.8967	0.897018		0.9025	do.	0.9025 (3)
6/5	1	0.9454	0.9455	0.945613		0.9512	do.	0.9512 (3)

(1) Hunter (1963)

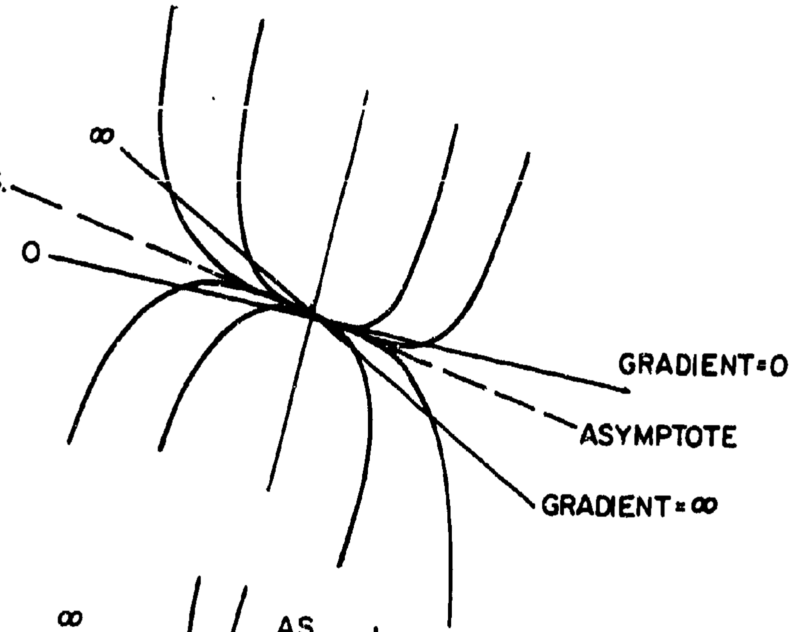
(2) node → saddle, α increasing

(3) saddle → node, α increasing

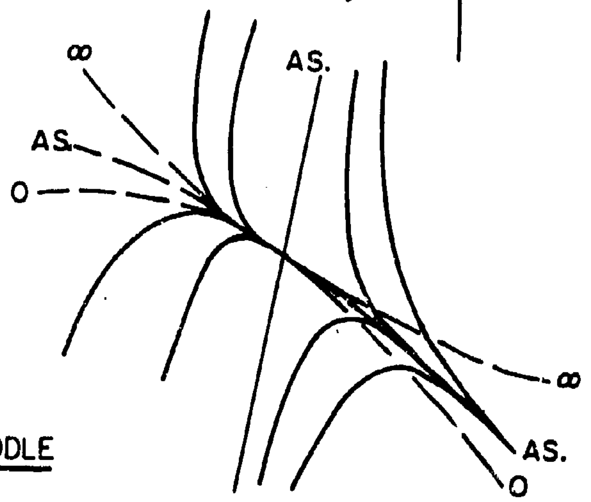
(4) node → spiral, α increasing

TABLE II

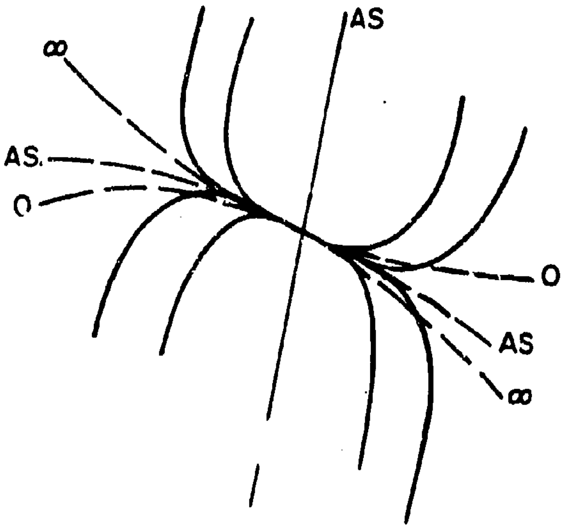
Entropy	γ	j	β_w	β_o	β_1	β_{exact}
varying	3	2	0.6667	0.5718	0.9750	1.0435
varying	3	1	0.6667	0.5570	0.7629	0.7738
constant	7	2	0.6269	1.6523	0.7733	
constant	7	1	0.6269	1.6198	1.0507	



FIRST ORDER NODE



SECOND ORDER NADDLE



THIRD ORDER NODE

FIG. 1

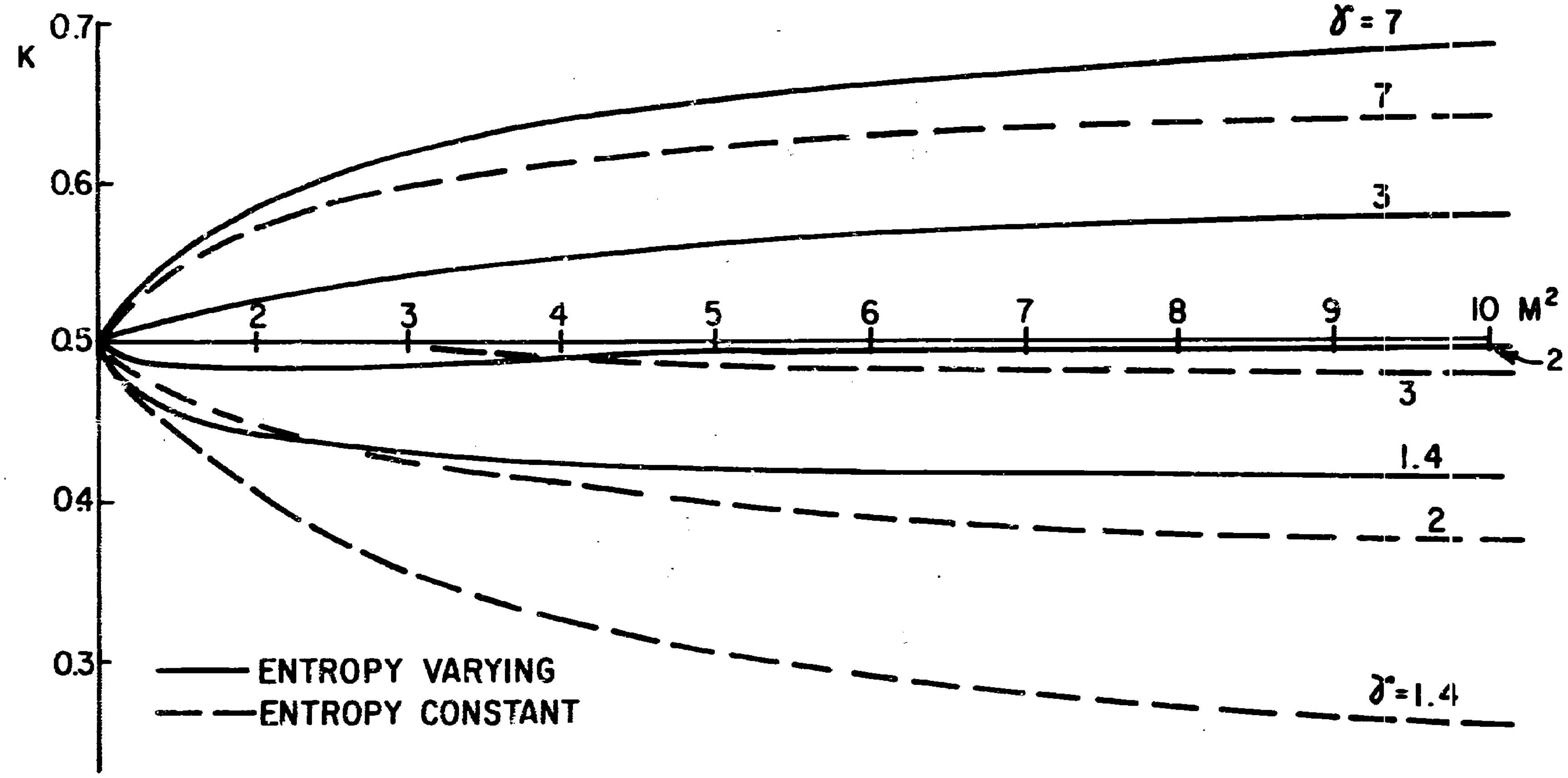
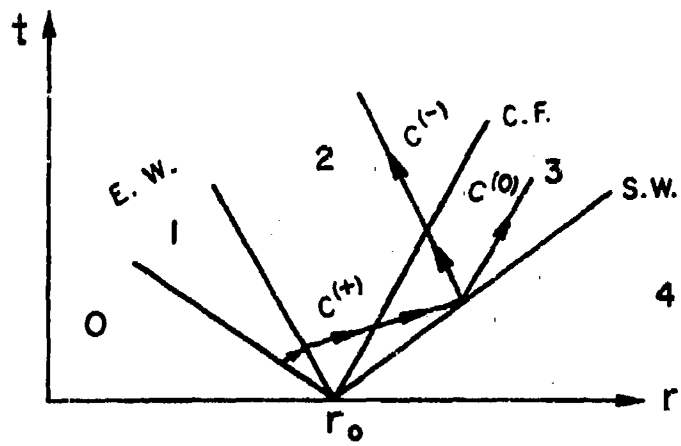

K
0.7
0.6
0.5
0.4
0.3
2
3
4
5
6
7
8
9
10 M²
γ = 7
7
3
2
3
1.4
2
γ = 1.4
ENTROPY VARYING
ENTROPY CONSTANT

FIG. 2



S.W. SHOCK WAVE
C.F. CONTACT FRONT
E.W. EXPANSION WAVE

FIG. 3

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	2b. GROUP

3. REPORT TITLE

COLLAPSING SPHERICAL SHOCK WAVES IN WATER

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

TECHNICAL REPORT

5. AUTHOR(S) (Last name, first name, initial)

Welsh, Robert L.

6. REPORT DATE September 1966	7a. TOTAL NO. OF PAGES 39	7b. NO. OF REFS 13

8a. CONTRACT OR GRANT NO. Nonr-222(79)	9a. ORIGINATOR'S REPORT NUMBER(S) AS-66-12
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

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13. ABSTRACT

The work presented here is related to various problems concerned with the propagation of shock waves, mainly imploding spherical and cylindrical shock waves, in water and (in part) in gases.

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KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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Spherical						
Cylindrical						
Self-Similar						
Implosions						
Ocean						

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